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B.A./B.Sc. FIRST SEMESTER EXAMINATION, JANUARY 2015

FIRST YEAR

MATHEMATICS (General)

Paper : I

Date : 14/01/2015

Time : 11 am – 2 pm

Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

(***Answer any five questions***) [5×5]

1. State De Moivre's theorem. Use it to prove that $\alpha^n+\beta^n=2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$, where α,β are the roots of the equation $x^2-2x+2=0$ and n is positive integer. [1+4]

2. Find the values of $(i-1)^{1/5}$. [5]

3. If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, form the equation whose roots are $\alpha+\beta,\ \beta+\gamma,\ \gamma+\alpha$. Also find the value of $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$. [4+1]

4. Solve the following cubic equation by Cardan's method: $x^3-18x-35=0$. [5]

5. a) Find the value of : $\begin{vmatrix} 13 & 37 & 73 \\ 19 & 43 & 79 \\ 15 & 39 & 75 \end{vmatrix}$. [2]

   b) Find the equation whose roots are greater than the roots of the equation $x^4+8x^3+x-5=0$ by 2. [3]

6. Show that : $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}=(a+b+c)^3$. [5]

7. a) If $A+I_3=\begin{bmatrix} 1 & 3 & 4 \\ -1 & 1 & 3 \\ -2 & -3 & 1 \end{bmatrix}$, evaluate $(A+I_3)(A-I_3)$, where I_3 represents 3×3 identify matrix. [3]

   b) Find the rank of the following matrix $\begin{bmatrix} 3 & 4 & -6 \\ 2 & -1 & 7 \\ 1 & -2 & 8 \end{bmatrix}$. [2]

8. Show that the system of equations : $x+y+z=6$, $x+2y+3z=14$, $x+4y+7z=30$ is consistent and hence solve it. [5]

Group – B

(***Answer any five questions***) [5×5]

9. a) Prove that the mapping $f:[0,\pi]\to\mathbb{R}$, defined by $f(x)=\sin x, x\in\mathbb{R}$ is neither injective nor surjective. [2]

   b) Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=2x+3, x\in\mathbb{R}$. Prove that f is bijective and find f^{-1}. [3]

10. a) In a group $(G,*)$ prove that $a*b=a*c$ implies b = c, where $a,b,c\in G$. [2]

    b) Consider the group $(\mathbb{Z},*)$ where '*' is defined by $a*b=a+b+1,\ \forall\ a,b\in\mathbb{Z}$ (the set of integers). Find the identity element in this group and find the inverse of $a\in\mathbb{Z}$. [3]

11. Prove that the set $S=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix}: a,b,c,d\in\mathbb{R}; ad-bc=1\right\}$ is a subgroup of the group of all 2×2 non-singular real matrices with respect to matrix multiplication. [5]

12. Assuming the set $T=\left\{a+b\sqrt{2}: a,b\in\mathbb{Q}\right\}$ as a ring with respect to usual addition and multiplication, prove that it is a field ($\mathbb{Q}$ is the set of rational numbers). [5]

13. Write down the matrix associated with the real quadratic form : $x^2+y^2+z^2+yz$. Find the eigen values of the matrix and hence determine the nature of the real quadratic form. [5]

14. Show that $A=\{(1,2,1),(0,1,0),(0,0,1)\}$ is a basis of $\mathbb{R}^3$. Express the vectors $(1,2,3)\in\mathbb{R}^3$ as a linear combination of the basis A. [2+3]

15. a) State the Cayley Hamilton Theorem. [1]

    b) Find the eigen values of the matrix $\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$ and one eigen vector corresponding to any one eigen value. [4]

16. a) Let $T=\left\{(x,y,z)\in\mathbb{R}^3; x^2+y^2=z^2\right\}$. Is T a subspace of the vector space $\mathbb{R}^3$? Justify your answer. [2]

    b) Let $\{\alpha,\beta,\gamma\}$ be a basis of a vector space V over a field F. Let $c\in F, c\neq 0$. Prove that $\{\alpha+c\beta, \beta, \gamma\}$ is a basis of V. [3]

Group – C

(*Answer any five questions*) [5×5]

17. a) Let f(x) be a real valued function. Find its domain of definition, where $f(x)=\sqrt{\log_e\left(\frac{5x-x^2}{4}\right)}$. [2]

    b) Examine whether $2x^3-12x^2+24x+6$ is increasing or decreasing on the real line. [3]

18. A function f(x) is defined as follows : $f(x)=\begin{cases} 5x-4 & \text{for } 0<x<1 \\ 4x^2-3x & \text{for } 1\le x<2 \\ 3x+4 & \text{for } x\ge 2, \end{cases}$

    examine the continuity at x = 1 and differentiability at x = 2 of f(x). [3+2]

19. If $y=\cos\left(m\sin^{-1}x\right)$, then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$, also find y_n for x = 0. [3+2]

20. If $u=\log_e(x^3+y^3+z^3-3xyz)$, then show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{-3}{(x+y+z)^2}$. [5]

21. State Euler's theorem on homogeneous function of two variables and verify this for the function $u(x,y)=\sin\left(\frac{x^2+y^2}{xy}\right)$. [1+4]

22. Find the condition that the curves $ax^2+by^2=1$ and $a'x^2+b'y^2=1$ intersect orthogonally. [5]

23. Show that the radius of curvature for the cycloid $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$ at $\theta=\frac{\pi}{3}$ is $2a\sqrt{3}$. [5]

24. Show that the pedal equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}-\frac{r^2}{a^2b^2}$. [5]

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